# KUVEMPU UNIVERSITY
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## TOPICS FOR INTERNAL ASSESSMENT ASSIGNMENTS (2019-20)
## Course: M.Sc Mathematics (Previous)

***Important Notes:*** *(1)Students are advised to read the separate enclosed instructions before beginning the writing of assignments.(2)Out of 20 Internal Assignment marks per Paper, 5 marks will be awarded for the regularity (attendance) to the Counseling/ Contact Programme classes pertaining to the paper. Therefore, the topics given below are only for 15 marks each paper. Answer all questions. Each question carries 05marks*

**PAPER I:ALGEBRA**

1. a)If p is a prime number and G is a group of order $p^n$, $n \geq 1$ then prove that the centre of G has at least 'p' elements.

   b) Let p be a prime dividing o(G). Show that every sylow p-subgroup of G/K is of the form PK/K, where P is a sylow p-subgroup of G.

   c)Prove that the product of any two ideals of a ring R is also an ideal of R. (2+2+1)

2. a)Define an Euclidean ring. Show that the ring I of all integers is a Euclidean ring.

   b) Let F be a field. If A = {(x, y, 0): x, y ∈ F}, B = {(0, y, z): y, z ∈F} be subspaces of $F^3(F)$, find the dimension of the subspace A+B.

   c)If W is a subspace of a finite dimensional vector space V, define the annihilator A(W) of a subspace W. Further show that

   i) $A(W_1 + W_2) = A(W_1) \cap A(W_2)$

   ii) $A(W_1 \cap W_2) = A(W_1) + A(W_2)$. (1+2+2)

3. a) Let T be a linear operator on a vector space V over F. If $W_1, W_2, \ldots, W_k$ are T-invariant subspaces of V, prove that $\sum_{i=1}^{k} W_i$ and $\bigcap_{i=1}^{k} W_i$ are T-invariant subspaces of V.

b) If f(x) ∈ F[x] is irreducible over F, then show that all its roots have the same multiplicity

**PAPER II:ANALYSIS-I**

1. a) Prove that $|x+y|^2 + |x-y|^2 = 2|x|^2 + 2|y|^2$, if $x \in R^k$ and $y \in R^k$. Interpret this geometrically, as a statement about parallelograms.

   b) Construct a bounded set of real numbers with exactly three limits points.

   c) Prove that every connected metric space with at least two points is uncountable..

2. a) Prove that every convex subset of $R^k$ is connected.

   b) Suppose $f$ is differentiable on $(0,\infty)$, $f''$ is bounded on $(0,\infty)$ and $f(x) \to 0$, as $x \to \infty$, then prove that $f'(x) \to \infty$ as $x \to \infty$.

   c) Suppose $f$ is bounded real function on $[a,b]$ and $f^2 \in \mathcal{R}$ on $[a,b]$. Does it follow that $f \in \mathcal{R}$? Does the answer change if we assume that $f^3 \in \mathcal{R}$?

3. a) Prove that let $\{f_n\}$ be uniformly bounded sequence of functions which are Riemannian integrable on $[a,b]$ and put $F_n = \int_a^x f_n(t)dt$, $a \leq x \leq b$ then there exists a subsequence $\{F_{n_k}\}$ Which converges uniformly on $[a,b]$.

   b) If $f(x) = 0$ for all irrational $x$, $f(x) = 1$ for all rational $x$ then prove that $f \notin \mathcal{R}$ on $[a,b]$ for any $a < b$.

**PAPER III:ANALYSIS-II**

1. a) Prove that every uniformly convergent sequence of bounded functions is uniformly bounded.
   b) Let $\{f_n\}_{n=1}^{\infty}$ be a sequence of continuous functions which converges uniformly to a function $f$ on a set $E$. Prove that $\lim_{n\to\infty} f_n(x_n) = f(x)$ for every sequence of points $x_n \in E$. such that $x_n \to x$ and $x \in E$. Is the converse of this true?
2. a) Consider $f(x) = \sum_{n=1}^{\infty} \frac{1}{1+n^2x}$. For what values of $x$ does the series converges absolutely? On what intervals does it converge uniformly? On what intervals does it fail to converge uniformly? Is $f$ continues wherever the series converges ? Is $f$ bounded.
   b) Consider $f(x) = \sum_{n=1}^{\infty} \frac{(nx)}{n^2}$, where $x$ is real. Find all discontinuous of $f$ and show that they form a countable dense set. Show that $f$ is nevertheless Riemann-integrable on every bounded interval.
   c) Let $x_n = 1 + \frac{1}{2} + \cdots + \frac{1}{n}$, and define $y_n = x_n - \log n$. Show that the sequence $(y_n)$ tends to a limit $y$. Where $0 < y \le 1$. Deduce that $1 - \frac{1}{2} + \frac{1}{3} - \cdots = \log 2$.
3. a) If the partial derivatives $f_x$ and $f_y$ exists and are bounded in a region $R \subset R^2$, then $f$ is continuous in $R$.
   b) If $f(0,0) = 0$ and $f(x,y) = \frac{xy}{x^2+y^2}$ if $(x,y) \neq (0,0)$. Prove that $(D_1f)(x,y)$ and $(D_2f)(x,y)$ exists at every point of $R^2$, although $f$ is not continuous at $(0,0)$
   c) Take $m = n = 1$ in the implicit function theorem and interpret the theorem graphically.

**PAPER IV:DIFFERENTIAL EQUATIONS**

1. a) Find the transformation which transforms $a_0(t)x'' + a_1(t)x' + a_2(t)x = 0$ into an equation whose in the first derivative term is absent.
   b) Show that the function $\{t^3, |t^3|\}$ are linearly independent on $[-1,1]$ but not on $[-1,0]$
2. a) Given a solution of $(1-t^2)x'' - 2tx' + 6x = 0, \phi_1(t) = 3t^2 - 1$. Find its general solution.
   b) Solve $x^{(4)} + 4x = 2\sin t + 4e^t + 1 + 3t^2$ by using the method of undetermined co-efficients.
3. a) Solve the nonlinear equation $p^2 - 3q^2 - u = 0$ with Cauchy data $u(x,0) = x^2$ using Cauchy method of characteristics.
   b) Find the solution of the heat equation of $u_t = c^2u_{xx}$ ; $0 < x < l$; $0 < t < \infty$ when subjected to the Neumann conditions $u(0,t) = k_1$, $u(l,t) = k_2$ ; and an initial condition $u(x,0) = \phi(x)$ for all $x$.

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## TOPICS FOR INTERNAL ASSESSMENT ASSIGNMENTS (2019-20)
## Course: M.Sc Mathematics (Final Year)

***Important Notes:*** *(1)Students are advised to read the separate enclosed instructions before beginning the writing of assignments.(2)Out of 20 Internal Assignment marks per Paper, 5 marks will be awarded for the regularity (attendance) to the Counseling/ Contact Programme classes pertaining to the paper. Therefore, the topics given below are only for 15 marks each paper. Answer all questions. Each question carries 05marks*

**PAPER V:COMPLEX ANALYSIS**

1. a) Show that there are complex numbers $z$ satisfying $|z-a|+|z+a| \leq 2|c|$ if and only if $|a| \leq |c|$. If this condition is fulfilled, what are the smallest and largest values of $|z|$?

   b) Find the radius of convergence of the following power series.

   i) $\sum \frac{z^n}{n!}$  ii) $\sum n!\, z^n$  iii) $\sum z^{n!}$

   c) If $\sum_0^\infty a_n$ converges, then show that $f(z)=\sum_0^\infty a_n z^n$ tends to $f(1)$ as $z$ approaches 1 in such a way that $\frac{|1-z|}{1-|z|}$ remains bounded.

2. a) Let $H(D)$ denote the set of all analytic functions defined on an open set $D$ and $f \in H(D)$. Let $z_0$ be any point of $D$ such that $f'(z_0) \neq 0$ then show that $f$ is conformal at $z_0$. Further give an example for isogonal mapping which is not conformal.

   b) Show that the cross ratio $(z_1, z_2, z_3, z_4)$ is real if and only if the four points lie on a circle or on a straight line.

   c) If the function $f(z)$ is analytic on a rectangle $R$, then show that $\int_{\partial R} f(z)\,dz = 0$.

3. a) State and prove the argument principle.

   b) Evaluate the following integrals.

   i) $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$  ii). $\int_0^\infty \frac{dx}{(x^2+1)(x^2+9)}$

   c) Show that non-constant entire functions are unbounded.

**PAPER VI: TOPOLOGY**

1. a) Let $X$ be the topological space. Suppose that $\mathcal{C}$ is a collection of open subsets of $X$ such that for each open set $U$ of $X$ and each $x$in $U$, there is an element $C$ of $\mathcal{C}$ such that $x \in C \subset U$. Then show that $\mathcal{C}$ is a basis for the topology of $X$.

   b) Let $\mathbb{R}_l$ denote $\mathbb{R}$ with lower limit topology and $\mathbb{R}_K$ denote $\mathbb{R}$ with $K$-topology. Then show that the topologies of $\mathbb{R}_l$and$\mathbb{R}_K$are not comparable.

2. a) Let $X$ and $Y$ be the topological spaces. Let $\pi_1$ and $\pi_2$ be the projections of $X \times Y$ onto $X$ and $Y$, respectively. Show that the collection

   $\mathcal{S} = \{\pi_1^{-1}(U) \mid U \text{ open in } X\} \cup \{\pi_2^{-1}(V) \mid V \text{ open in } Y\}$

   is a subbasis for the product topology on $X \times Y$.

   b) Let $X$ be the topological space and $A, B \subset X$. Then prove the following.

   i) If $A \subset B$ then $A^0 \subset B^0$  ii) $(A \cap B)^0 = A^0 \cap B^0$  iii). $A^0 \cup B^0 \subset (A \cup B)^0$.

   Further give an example to show that equality does not hold good.

c) Prove that for functions $f: \mathbb{R} \to \mathbb{R}$, the $\epsilon$-$\delta$ definition of continuity implies the open set definition.

3. a) Show that a path connected space $X$ is necessarily connected. Is converse true? Justify.

b) Show that every compact subspace of a metric space is bounded in that metric and is closed. Find a metric space in which not every closed bounded subspace is compact.

c) Show that every well-ordered set $X$ is normal in the order topology.

**PAPER VII: MEASURE THEORY & FUNCTIONAL ANALYSIS**

1. a) Prove that the interval $(a, \infty)$ is measurable. Deduce that every Borel set is measurable.

b) Prove that a bounded function defined on a set of finite measure is Lebesgue integrable if and only if it is measurable.

c) Define the Lebesgue integral. Consider $(x) = \sum_{n=1}^{200} \frac{1}{n^6} \chi_{\left[0, \frac{n}{200}\right]}(x), x \in [0,1]$, where $\chi$ is the characteristic function. Find the Lebesgue integral of $f$ on [0,1].

2. a) Let $f \geq 0$ and measurable. Show that $\exists$ a sequence $\{\varphi_n\}$ of simple functions $\ni$ $\varphi_n \uparrow f$.

b) Define a complete metric space. Show that C[a, b] is not complete under integral metric.

c) Prove that a metric space is compact if and only if it is sequentially compact.

3. a) If X is a finite dimensional normed linear space, then prove that any two norms on X are equivalent. Does the converse hold true? Justify.

b) Is there any result which guarantees the uniqueness of the norm attaining point of a bounded linear operator? Discuss.

c) Let X be a Banach space. Prove that X is reflexive if and only if $X^*$ is reflexive.

**PAPER VIII: NUMERICAL ANALYSIS**

1. a) Use Bairstow's method to extract a quadratic factor of the form $x^2 - px - q$ from a polynomial Choose $p = 1, q = 2$.

b) Explain the Householder method to reduce the following matrix in to tri-diagonal form

$$A = \begin{pmatrix} 1 & 4 & 0 \\ 4 & 1 & -4 \\ 0 & -4 & 2 \end{pmatrix}$$

2. a) Fit both linear and quadratic curve for a function $f(x) = \frac{x}{x+1}$ over an interval *[0, 1]* with respect to the weight function *w(x) = 1* using least square approximation method.

b) Find the cubic spline interpolation polynomial in the interval [0,4] for the following data:

| | | | | | |
|---|---|---|---|---|---|
| *x* | *0* | *1* | *2* | *3* | *4* |
| *y* | *3* | *3* | *9* | *27* | *63* |

Given $s''(0) = s''(4) = 0$.

3. a) Solve an initial value problem $\frac{dy}{dx} = \frac{2x}{1+y}$, $y(0) = 0$ in the range $0 \leq x \leq 1$ using Milne's Predictor-Corrector method, choose $h = 0.2$.

b) Find the solution of a BVP $y'' - 2y' + 4y = \sin x$ $y(0) = 0, y(1) = 0$ using finite difference method, choose $h = 0.25$.

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